Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

UNIVERSITY OF BIRMINGHAM

School of Computer Science

Theories of Computation

Main Summer Examinations 2024

Time allowed: 2 hours

[Answer all questions]

Turn Over

Note

Answer ALL questions. Each question will be marked out of 20. The paper will be marked out of 60, which will be rescaled to a mark out of 100.

Question 1

(a) Consider the following context-free grammar for simple Java methods:

⇒	Method	::= Visibility Modifier Type Word (Type Word) { Operation ; }
	Visibility	::= `public` | `private`
	Modifier	::= ε | `static` | `abstract`
	Type	::= `void` | `int` | `Boolean` | `String` | Type`[]`
	Word	::= [a-z] | Word Word
	Number	::= [0-9] | Number Number
	Value	::= "Word" | Number | Number + Number
	Operation	::= `print(`Value`)` | `return` Value

(i) For both of the following two simple Java methods, state whether or not they are generated by the above grammar (ignoring whitespace such as spaces).

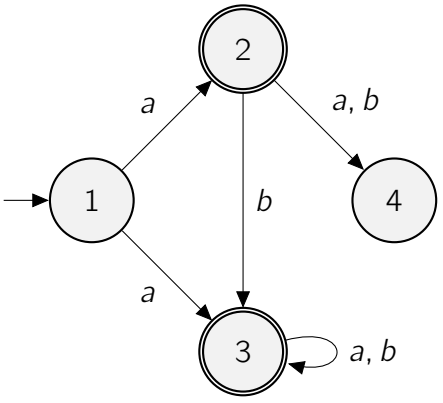
i. `private abstract int add5(int x) { return x + 5; }` **[1 mark]**

ii. `public static void main(String[] args) { print("hi"); }` **[1 mark]**

(ii) Write the leftmost derivation of the following simple Java method. At each stage, write a dot over the variable that will be replaced in the next step.

`public String ln(Boolean x) { return 0; }` **[4 marks]**

(b) Consider the following nondeterministic finite automaton (NFA), which is defined on the alphabet $\Sigma = \{a, b\}$:



(i) Write a regular expression for the language of this NFA. **[3 marks]**

(ii) Draw the partial DFA that results from determinising the NFA. **[3 marks]**

(iii) Under what conditions is a total DFA said to be *minimal*? **[3 marks]**

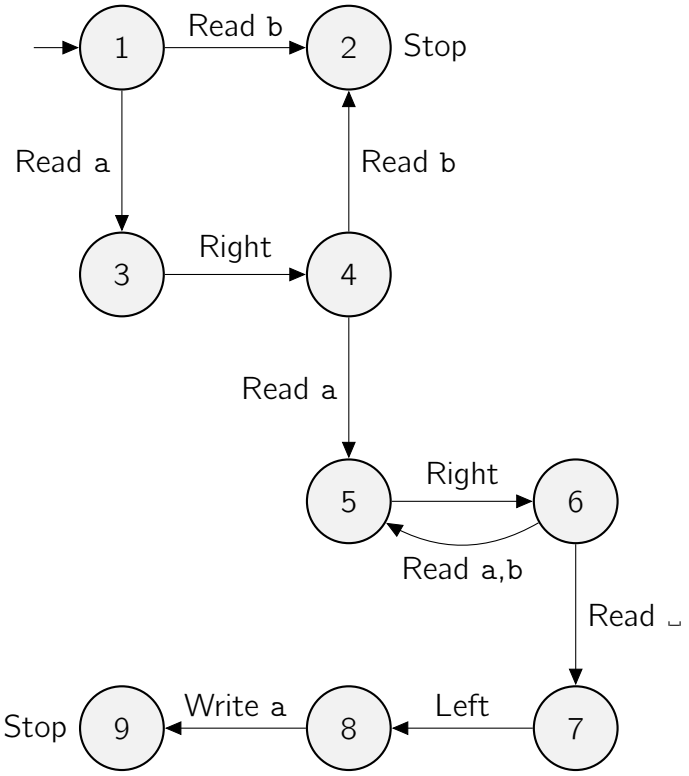
(c) Adira has written a sorting program A, which sorts any list of natural numbers. Let $T(n)$ be the time in milliseconds that A takes to sort a list of length $n \in \mathbb{N}$. Adira finds that:

- Sorting the empty list always takes 1 millisecond,
- Sorting any list of one element always takes 2 milliseconds,
- For any $n \in \mathbb{N}$, sorting any list of length $n+2$ always takes $2T(n) + T(n+1)$ milliseconds.

Using course-of-values induction, prove that when asked to sort any list of length $n \in \mathbb{N}$ the program A always takes 2^n milliseconds. **[5 marks]**

Question 2

Consider the following Turing machine $\mathcal{M}$ on alphabet $\Omega = \{\texttt{a}, \texttt{b}, \textvisiblespace\}$ with return value set $V = \{\texttt{Done}\}$.



(a) Give the complete run of machine $\mathcal{M}$ above on the word aab, with the head positioned on the leftmost a.

At each step, indicate the tape contents, the position of the head, the current state and the instruction (including the result if it is a Read). **[6 marks]**

Hint: No more than 12 steps are required.

(b) Now suppose the tape initially contains a block of a's and b's of length $n \geq 2$, on an otherwise blank tape, with the head on the leftmost non-blank character. State, with justification, the number of steps (including Stop) required in:

(i) The best case. **[2 marks]**

(ii) The worst case. **[2 marks]**

(iii) The average case. **[4 marks]**

Note: Assume a and b are equally likely and the characters of the input are independent.

(c) Given two programs A and B, we build the program AB which first runs A and then B on the same input of length n. The worst case running time of A is given by a

function $T(n)$, which is $n^7 + 5$ for $n < 1000$ and $n^3 + 5$ for $n \geq 1000$. The worst case running time of B is given by a function $R(n)$, which is $O(n \log_{10} n)$. Show carefully that the worst case running time of AB is $O(n^3)$. **[6 marks]**

Note: You may assume that $\log_{10} n < n$ for all $n > 0$.

Question 3

(a) What does it mean for a property of words to be in NP? **[5 marks]**

(b) The *factorisation* decision problem is as follows: given two integers $a, b > 1$, written in binary, say whether a has a factor less than b. Explain informally why this problem is in NP. **[5 marks]**

(c) Andrew is the manager of GrammarMe Ltd. Every Monday, he receives two context free grammars from the software team and has to check that precisely one of them is ambiguous (he does not need to specify which one).

(i) Can he write a Java program to do this? Explain your answer. (You may use the fact that ambiguity of context free grammars is an undecidable property.) **[6 marks]**

(ii) What if Andrew uses a different programming language? **[4 marks]**

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.